Title: One Notion, Many Generalizations: The Story of Chordality in Higher Dimensions

Abstract: Chordal graphs occupy a central position in discrete mathematics due to the remarkable convergence of several seemingly unrelated ideas: induced cycle structure, elimination orderings, clique decompositions, efficient algorithms, and connections with commutative algebra through Fröberg's theorem on linear resolutions of edge ideals. Over the last two decades, this rich interplay has inspired numerous attempts to extend the notion of chordality from graphs to hypergraphs and simplicial complexes.

In higher dimensions, however, the classical equivalences break apart. Different communities — including combinatorics, topology, commutative algebra, and theoretical computer science — have proposed distinct notions of "higher chordality," each preserving different aspects of the graph-theoretic theory. These include approaches based on simplicial elimination, acyclicity, collapsibility, shellability, and homological properties of monomial ideals. The resulting landscape is diverse, subtle, and still actively evolving.

This talk will present a broad overview of these developments from a unifying perspective. Beginning with classical chordal graphs and Fröberg's theorem, I will discuss several influential notions of chordal hypergraphs and higher-dimensional chordality, emphasizing the motivations behind them and the mathematical phenomena they capture. Along the way, I will highlight interactions between combinatorics, topology, and commutative algebra, as well as emerging algorithmic and structural questions related to recognition problems, decomposition methods, and higher-dimensional analogs of graph-theoretic properties.

The aim of the talk is not to survey all existing definitions exhaustively, but rather to illustrate how a single classical notion gives rise to many different and fascinating theories in higher dimensions.